THEORY OF A FREE-FIELD CONDUCTION PROPULSION UNIT

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UDC 538.4

A free-field conduction-type MHD propulsion unit is one of the promising types [1]. The operating principle can be understood from an ideal model in the form of an infinitely long cylinder placed in an immobile conducting liquid. The magnetic field is set up by a current distributed over the surface of the cylinder $i_z(\alpha) = i_0 \sin m\alpha$ (r, α, and z are cylindrical coordinates and m is an integer). The electric current in the liquid is supplied by electrodes also distributed along the surface, with the potential distribution on the electrodes in the form $\varphi(\alpha) = \varphi_0 \sin m\alpha$; then the liquid contains mutually perpendicular electric and magnetic fields **E** and **H** shown in Fig. 1, which set up a bulk force in the liquid $\mathbf{f} = (1/c)\cdot[\mathbf{j} \times \mathbf{H}]$, which acts along the z axis. The cylinder experiences a force in the opposite direction.

Some engineering developments [1, 2] of such propulsion units are based on magnetohydrodynamic examination of ideal models, and the results have appeared in subsequent publications [3, 4].

It has been shown [5] for an induction-type MHD propulsion unit that there are major deviations from the ideal results if the length is finite. A method has been proposed for improving the efficiency by amplitude modulation.

Interest attaches to an analogous study for a conduction device.

1. As in [5], the energy characteristics are examined on a model in the form of a flat plate of finite width. We consider the variational problem for the optimum potential distribution over the width of the plate that provides maximum efficiency for a given magnetic-field distribution.

We consider a plate with width a on the x axis and infinitely extended along the y axis, which is in an unbounded conducting liquid of conductivity σ and density ρ, and which is set into motion in its plane along the negative direction of the x axis on account of electromagnetic forces. The magnetic field is produced by surface currents in the plane of the plate that are periodic along the y axis with period λ (Fig. 2):

$$\mathbf{i}(x, y) = [i_1(x)\mathbf{e}_x + i_2(x)\mathbf{e}_y]\,e^{ik_1y}, \quad k_1 = 2\pi/\lambda \tag{1.1}$$

(here and subsequently the symbols denoting the surface currents will be given either with subscripts or arguments in order to avoid confusion with the imaginary unit i).

The surfaces of the plate (both sides) are ideally sectioned electrodes that have the potential distribution

$$\Phi(x)\,e^{ik_1y} \quad (|x| \leqslant a/2).$$

The equation of continuity $\partial i_x/\partial x + \partial i_y/\partial y = 0$ gives us from (1.1) that

$$i_2(x) = \frac{i}{k_1}\frac{di_1(x)}{dx}, \tag{1.2}$$

and therefore the current distribution $\mathbf{i}(x, y)$ over the plate is completely determined by the $i_1(x)$.

2. To determine the electric field **E** and the magnetic field **H** in the liquid, we need to know the velocity distribution **v**. We assume that the velocity pattern around the plate, which is set in motion by the electromagnetic forces, does not differ from that in flow around a classical plate. This is so if

$$N = \frac{\sigma H_0^2 a}{c^2 \rho u_0} \ll 1, \tag{2.1}$$

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 109-118, September-October, 1980. Original article submitted December 17, 1979.

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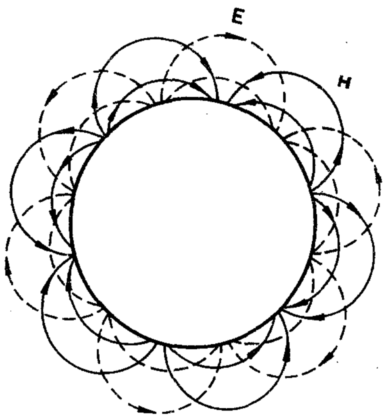

Fig. 1

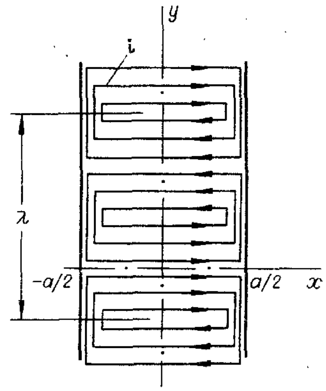

Fig. 2

where N is the MHD interaction parameter; H_0, maximum magnetic field strength; u_0, plate speed; and c, speed of light.

We also assume that the magnetic Reynolds number is small:

$$\mathrm{Re}_m = 4\pi\sigma u_0 a/c^2 \ll 1, \tag{2.2}$$

which on account of the low conductivity of seawater ($\sigma = 5\cdot 10^{10}$/sec) is correct within fairly wide ranges in a and u_0. On the basis of (2.2), we can take the magnetic field in the liquid as the field set up by the currents of (1.1). The latter is found by means of Fourier transformation and takes the following form for z > 0:

$$\mathbf{H} = -\mathrm{grad}\, \psi(x, y, z); \tag{2.3}$$

$$\psi = -\frac{2\pi i}{ck_1} e^{ik_1 y} \int_{-\infty}^{\infty} i_1(k)\, e^{ikx - \sqrt{k^2 + k_1^2}\, z} dk,$$

$$i_1(k) = \frac{1}{2\pi} \int_{-a/2}^{a/2} i_1(x)\, e^{-ikx}\, dx. \tag{2.4}$$

H is defined by the following in the half-space z < 0:

$$H_x(-z) = -H_x(z),\ H_y(-z) = -H_y(z),\ H_z(-z) = H_z(z),$$

while in the plane z = 0 there are discontinuities in H_x and H_y for $|x| \leqslant a/2$ on account of the currents of (1.1).

The electric-field potential satisfies the following equation [6] in a coordinate system coupled to the plate for a homogeneous liquid with a scalar conductivity σ = const subject to (2.2):

$$\Delta\Phi = (1/c)\mathbf{H}\,\mathrm{rot}\,\mathbf{v} = (1/c) H_y \partial v_x/\partial z, \tag{2.5}$$

where $\mathbf{v} = v_x(x, z)\mathbf{e}_x$ is the velocity field. The right side of this equation describes the distribution of the space-charge density ρ_e in the liquid:

$$\rho_e = -\frac{1}{4\pi c} H_y \frac{\partial v_x}{\partial z}.$$

We do not know the exact expression for the velocity distribution $v_x(x, z)$ within the boundary layer for a plate of finite width in a flow having a large Reynolds number; however, if we restrict consideration to a thin boundary layer, i.e., to the case

$$\max \Delta \ll \min(\lambda, a) \tag{2.6}$$

where $\Delta(x)$ is the thickness of the boundary layer, we do not need to know $v_x(x, z)$ precisely in order to determine Φ. The distribution of ρ_e in the boundary layer in (2.6) may be referred to a surface distribution, whose density Σ is independent of the velocity profile in the boundary layer. In fact, if (2.6) applies, H_y varies only slightly across the boundary layer, and

$$\Sigma = 2\int_0^{\Delta(x)} \rho_e(x, z)\,dz = -\frac{2}{4\pi c}\int_0^{\Delta(x)} H_y(x, y, z)\frac{\partial v_x(x, z)}{\partial z}dz = -\frac{u_0}{2\pi c}H_y\Big|_{z=+0} \quad (2.7)$$

(here we have used the fact that the velocity of the liquid outside the boundary layer is given by (2.1) as u_0), and (2.5) becomes

$$\Delta\Phi = (2u_0/c)H_y|_{z=+0}\delta(z), \quad (2.8)$$

where $\delta(z)$ is a delta function and the factor 2 corresponds to charge on both sides of the plate.

Two possibilities occur on substituting the boundary conditions. Firstly, we can specify the potential distribution at the electrodes (i.e., in the band $|x| \leqslant a/2$; $z = 0$); then one determines the potential outside the plate by integrating (2.8) in elliptical coordinates in the (x, y) plane, with the results expressed in terms of Mathier functions, which hinders analysis of the integral characteristics. The problem is simplified if we note that the z component of the electric field in the plane $z = 0$ is zero outside the plate for any potential distribution on the electrodes. This enables us to reduce the problem to one for the half space $z > 0$ with the following conditions: at the boundary $z = 0$

$$-\frac{\partial\Phi}{\partial z}\Big|_{z=+0} = E(x)\,e^{ik_1y}, \quad E(x) = \begin{cases} E_0(x) & \text{for} \quad |x| \leqslant a/2, \\ 0 & \text{for} \quad |x| > a/2 \end{cases} \quad (2.9)$$

and at infinity

$$\Phi|_{z=\infty} = 0.$$

The function $E_0(x)$ in (2.9) characterizes the distribution of E_z on the plate on the positive z side (E_z has the opposite sign on the other side). The potential distribution on the electrodes that provides this $E_0(x)$ is found from the solution. Note that $E_z|_{z=+0}$ can be represented as the sum of the electric field $2\pi\Sigma$ due to the surface charges Σ induced by the motion of the liquid and the field $E_{1z}|_{z=+0} = E_1(x)e^{ik_1y}$ set up by the electrodes in the immobile liquid. Then from (2.3), (2.4), and (2.7) we have

$$E_0(x) = \frac{2\pi u_0}{c^2}\int_{-\infty}^{\infty} i_1(k)\,e^{ikx}dk + E_1(x) \quad (|x| \leqslant a/2). \quad (2.10)$$

The problem of (2.8) and (2.9) is solved by the standard Fourier transformation method. We put

$$E(x) = \int_{-\infty}^{\infty} E(k)\,e^{ikx}dk,$$

and write the solution for $z > 0$ as

$$\Phi(x, y, z) = e^{ik_1y}\int_{-\infty}^{\infty}\frac{E(k)\,e^{ikx-\sqrt{k^2+k_1^2}z}}{\sqrt{k^2+k_1^2}}dk. \quad (2.11)$$

3. We calculate some integral quantities: the thrust acting on the plate and the input electrical power. The force F and power Q are referred to unit length of the plate along the y axis, where the force is equal in magnitude to the total force acting on the corresponding volume of liquid, while the sense is the opposite, i.e.,

$$\langle F_x\rangle = -2\int_0^{\infty}\int_{-\infty}^{\infty}\langle f_x\rangle\,dxdz.$$

The input power is

$$\langle Q\rangle = 2\int_0^{\infty}\int_{-\infty}^{\infty}(\mathbf{E}\cdot\mathbf{j})\,dxdz$$

and goes to performing mechanical work on the liquid and to Joule heating. Here the $\langle\ \rangle$ denote averaging with respect to y; $\mathbf{f} = (1/c)[\mathbf{j} \times \mathbf{H}]$ is the bulk force density.

On the assumption of (2.6) we neglect the volume occupied by the boundary layer and calculate the current density for the entire space as $\mathbf{j} = \sigma[\mathbf{E} + (u_0/c)(\mathbf{e}_z \times \mathbf{H})]$; the result from using (2.11), (2.3), and (2.4) is

$$\langle F_x\rangle = -\frac{2\pi^2\sigma}{c^2}\,\mathrm{Re}\int_{-\infty}^{\infty}\frac{2E(k)\,i_1^*(k) - \dfrac{2\pi u_0}{c^2}\,i_1(k)\,i_1^*(k)\left(1+\dfrac{k^2+k_1^2}{k_1^2}\right)}{\sqrt{k^2+k_1^2}}\,dk,$$

$$\langle Q\rangle = 2\pi\sigma\,\mathrm{Re}\int_{-\infty}^{\infty}\frac{E(k)\,E^*(k) - \left(2\pi u_0/c^2\right)E(k)\,i_1^*(k)}{\sqrt{k^2+k_1^2}}\,dk.$$

From (2.10)

$$E(k) = (2\pi u_0/c^2)i_1(k) + E_1(k),$$

and the result can be simplified somewhat:

$$\langle F_x\rangle = \frac{2\pi^2\sigma}{c^2}\,\mathrm{Re}\int_{-\infty}^{\infty}\frac{2E_1(k)\,i_1^*(k) - \dfrac{2\pi u_0}{c^2}\,i_1(k)\,i_1^*(k)\,k^2/k_1^2}{\sqrt{k^2+k_1^2}}\,dk; \tag{3.1}$$

$$\langle Q\rangle = 2\pi\sigma\,\mathrm{Re}\int_{-\infty}^{\infty}\frac{E_1(k)\,E_1^*(k) + \left(2\pi u_0/c^2\right)E_1(k)\,i_1^*(k)}{\sqrt{k^2+k_1^2}}\,dk. \tag{3.2}$$

For convenience in analyzing the result we put it in dimensionless form; let the dimension a, the maximum current i_x (denoted by i_0), and $H_0 = 2\pi i_0/c$, u_0H_0/c be the corresponding scales. We retain the previous symbol for the dimensionless coordinate x and put $i_1(x)$, $E_1(x)$ as

$$i_1 = i_0 i(x),\ \ E_1 = (u_0H_0/c)e_1(x).$$

Then the Fourier components will be

$$i_1(k) = i_0 a i(q), \quad E_1(k) = (u_0H_0/c)\,a e_1(q),$$

$$\begin{vmatrix} e_1(q) \\ i(q) \end{vmatrix} = \frac{1}{2\pi}\int_{-1/2}^{1/2}\begin{vmatrix} e_1(x) \\ i(x) \end{vmatrix} e^{-iqx}dx. \tag{3.3}$$

We substitute (3.3) into (3.1) and (3.2) to get

$$\langle F_x\rangle = -\left(2\pi\sigma u_0 H_0^2 a^2/c^2\right)F_1, \quad \langle Q\rangle = \left(2\pi\sigma u_0^2 H_0^2 a^2/c^2\right)Q_1; \tag{3.4}$$

$$F_1 = \mathrm{Re}\int_{-\infty}^{\infty}\frac{e_1(q)\,i^*(q) - \left(q^2/2q_0^2\right)i(q)\,i^*(q)}{\sqrt{q^2+q_0^2}}\,dq,$$

$$Q_1 = \mathrm{Re}\int_{-\infty}^{\infty}\frac{e_1(q)\,e_1^*(q) + e_1(q)\,i^*(q)}{\sqrt{q^2+q_0^2}}\,dq. \tag{3.5}$$

Here $q_0 = k_1 a = (2\pi/\lambda)a = n\pi$ is the dimensionless wave number, which defines the number $n = a/(\lambda/2)$ of half-waves in the current of (1.1) within dimension a.

The efficiency is

$$\eta = |\langle F_x\rangle|u_0/\langle Q\rangle = F_1/Q_1$$

and is the ratio of the dimensionless force to the dimensionless power.

4. We now determine the distribution of the normal component of the electric field that provides the maximum efficiency. Here we consider the following variational problem: with a given H_0 and $i(x)$, which determines the distribution of the magnetic field, we have to find the optimum $e_1(x)$ which minimizes $\langle Q\rangle$ while providing the necessary thrust $\langle F_x\rangle$. The necessary thrust is determined by the resistance force, i.e., $|\langle F_x\rangle| = c_f \times 2a\rho u_0^2/2$, and then from (3.4) and the definition of (2.1) we have

$$F_1 = c_f/2\pi N. \tag{4.1}$$

Therefore, $e_1(x)$ must provide minimum Q_1 subject to condition (4.1) on F_1; the problem is therefore solved with an undetermined Lagrange multiplier, and the condition $\delta(Q_1 + \lambda_1 F_1) = 0$ for the functionals of (3.5) gives

$$\operatorname{Re}\int_{-\infty}^{\infty} \frac{\delta e_1(q)\left[2e_1^*(q) + (1+\lambda_1)\, i^*(q)\right]}{\sqrt{q^2+q_0^2}}\, dq = 0 \tag{4.2}$$

(λ_1 is the undetermined Lagrange multiplier). Without loss of generality, the variation in the function may be taken as $\delta e_1(x) = \varepsilon\delta(x - x_0)$, where $|x_0| < 1/2$; we then get the Fourier components as $\delta e_1(q) = (\varepsilon/2\pi)e^{-iqx_0}$ and substitute the latter into (4.2) on the basis that ε is in general a complex number to get

$$\int_{-\infty}^{\infty} \frac{2e_1(q) + (1+\lambda_1)\, i(q)}{\sqrt{q^2+q_0^2}}\, e^{iqx_0} dq = 0.$$

This equation must apply for all x_0 that satisfy $|x_0| < 1/2$; also, $e_1(x)$ and $i(x)$ are identically zero for $|x_0| \geqslant 1/2$, which means that

$$2e_1(q) + (1+\lambda_1)i(q) = 0,\ \text{i.e.,}\ e_1(x) = (1/\gamma)i(x),\ \gamma = \text{const.}$$

Therefore, the optimum system has the distribution of E_{1z} over the width proportional to i_x; the overall z component of the electric field is then given by (2.10) as also proportional to i_x, and the dimensionless analog of $e_1(x)$, viz., $e(x)$, for the total field is

$$e(x) = (1 + 1/\gamma)i(x). \tag{4.3}$$

This means that the space charge created by the flow of the liquid sets up an electric field that strengthens the initial field created by the electrodes in the immobile liquid. The maximum value of $i(x)$ over the segment $-1/2 \leqslant x \leqslant 1/2$ is 1, so the factor $1/\gamma_1 = 1 + 1/\gamma$ defines the scale of the electric field in the system:

$$E_0 = (1 + 1/\gamma)u_0H_0/c.$$

Consequently, $\gamma_1 = \gamma/(\gamma + 1)$ is the load parameter ($\gamma_1 = u_0H_0/cE_0$), which determines the efficiency of this system.

We now determine the potential distribution on the plate (on the electrodes) for the optimum system. This follows from (2.11) and (4.3) as

$$\Phi(x, y, 0) = \frac{1}{\gamma_1}\frac{u_0H_0}{c}\, a e^{iq_0y}\varphi(x), \quad \varphi(x) = \int_{-\infty}^{\infty} i(q)\left(\sqrt{q^2+q_0^2}\right)^{-1} e^{iqx}dq. \tag{4.4}$$

5. Then the optimum system has the following dimensionless force, power, and efficiency, which are denoted by F_0, Q_0, η_0:

$$F_0 = \frac{1-\gamma_1}{\gamma_1}I_1(q_0) - \frac{1}{2}I_2(q_0), \quad Q_0 = \frac{1-\gamma_1}{\gamma_1^2}I_1(q_0),$$

$$\eta_0 = \gamma_1\left[1 - \frac{\gamma_1}{2(1-\gamma_1)}\frac{I_2(q_0)}{I_1(q_0)}\right], \tag{5.1}$$

$$I_1(q_0) = \int_{-\infty}^{\infty}\left(\sqrt{q^2+q_0^2}\right)^{-1}|i(q)|^2 dq, \quad I_2(q_0) = \int_{-\infty}^{\infty}\left(\sqrt{q^2+q_0^2}\right)^{-1}(q^2/q_0^2)\,|i(q)|^2 dq.$$

This shows that the integral I_2 diverges in the case $i(x) = 1$ ($i_x = i_0 = \text{const}$ over the width of the plate) when $i(q) \sim \sin(q/2)/(q/2)$; the electromagnetic field sets up an infinite resistance force instead of a thrust. Physically this is explained by the closing currents i_y being localized at the edges of the plate in accordance with (1.2), i.e.,

$$i_2(x) = \frac{i}{k_1}i_0\left[\delta\left(x - \frac{a}{2}\right) - \delta\left(x + \frac{a}{2}\right)\right].$$

The magnetic field set up by these currents at the edge of the plate is infinite, and this is responsible for the infinite resistance.

In practice, of course, the distribution of i_x must be such that i_y does not exceed i_0. For this purpose we put $i(x)$ as

$$i(x) = \frac{\left(1 - e^{-(0.5+x)q_0}\right)\left(1 - e^{-(0.5-x)q_0}\right)}{\left(1 - e^{-0.5q_0}\right)^2} \quad (|x| \leqslant 1/2). \tag{5.2}$$

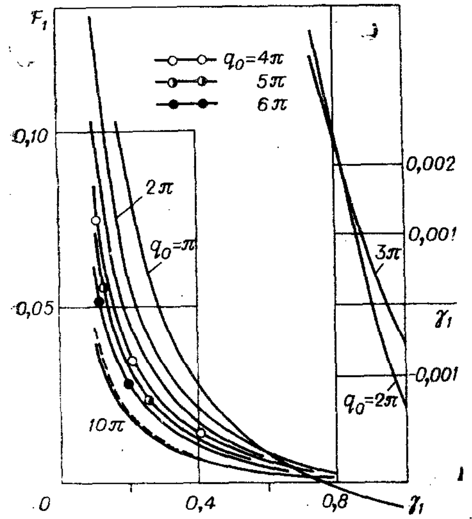

Fig. 3

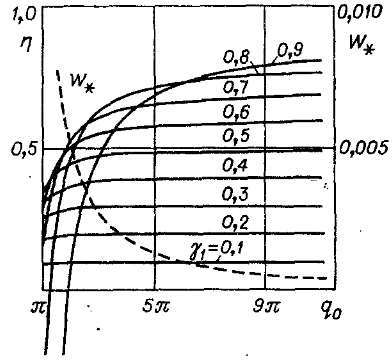

Fig. 4

Then we have $i(x) = 1$ over almost all the width of the plate, apart from the ends of the range $[-1/2, 1/2]$; $i(x)$ falls to zero at the ends of the range over segments of order $\Delta_1 = 1/q_0$. Then (5.2) meets the condition $|i_2| < i_0$, and the modulus of the magnetic field does not exceed H_0 at any point. The distribution of (5.2) corresponds to the Fourier transform of $i(q)$, which decreases as $1/q^2$ for $|q|$ large; consequently, the intergrals I_1 and I_2 converge.

As the main peak in the power spectrum of $|i(q)|^2$ has a width of order π, the integrals I_1 and I_2 appearing in (5.1) may be expanded as asymptotic series in powers of $1/q_0$. Parseval's inequality

$$\int\limits_{-\infty}^{\infty} |i(q)|^2 dq = (1/2\pi) \int\limits_{-1/2}^{1/2} |i(x)|^2 dx, \quad \int\limits_{-\infty}^{\infty} |i(q)|^2 q^2 dq = -(1/2\pi) \int\limits_{-1/2}^{1/2} i''(x)\, i^*(x) ax$$

enable us to calculate the coefficients to $1/q_0$ and $1/q_0^2$, and the result for the $i(x)$ defined by (5.2) is

$$I_1 = \frac{1}{2\pi}\left[\frac{1}{q_0} - \frac{1}{2q_0^2} + O\left(\frac{1}{q_0^3}\right)\right], \quad I_2 = \frac{1}{2\pi}\left[\frac{1}{q_0^2} + O\left(\frac{1}{q_0^3}\right)\right].$$

Then

$$F_0 = \frac{1}{2\pi q_0}\frac{1}{\gamma_1}\left[(1-\gamma_1) - \frac{1}{2q_0} + O\left(\frac{1}{q_0^2}\right)\right],$$
$$\eta_0 = \gamma_1\left[1 - \frac{\gamma_1}{2(1-\gamma_1)}\frac{1}{q_0} + O\left(\frac{1}{q_0^2}\right)\right], \qquad (5.3)$$

where the principal terms in the expansion correspond to an infinite plate (more precisely, part of the plate of width a conceptually cut from an infinite plate with $i(x)$, $e_1(x)$ constant along x), i.e.,

$$F_\infty = (1/2\pi q_0)(1-\gamma_1)/\gamma_1, \quad \eta_\infty = \gamma_1. \qquad (5.4)$$

We see from (5.3) that the end-effects have a marked influence for $\gamma_1 \to 1$; in fact, (5.4) shows that the system works as a propulsion unit for $0 < \gamma_1 < 1$ if we neglect the end-effects, i.e., $F > 0$, $Q > 0$). We see from (5.1) that the end-effects cause the thrust to become zero for $\gamma_1 = \gamma_0(q_0) < 1$, and for $\gamma_0 < \gamma_1 < 1$ the system acts as an electromagnetic brake (since then $Q_0 > 0$). Only for $\gamma_1 > 1$ does the system act as a generator, as in an infinite system.

Results for F_0 from (5.1) are shown in Fig. 3; for $q_0 = \pi$ we have $\gamma_0 \approx 0.75$, while for larger q_0 the value of γ_0 approaches one, and the range $[\gamma_0, 1]$ of values of γ_1 for braking action becomes narrower. The maximum value of q_0 in Fig. 3 is taken as 10π. The asymptotic formulas of (5.3) can be used for $q_0 > 10$ for the propulsion range of practical interest with $0 < \gamma_1 < 0.9$; for clarity, the broken line in Fig. 3 shows the asymptotic curve for $q_0 = 10\pi$; clearly, the difference is slight.

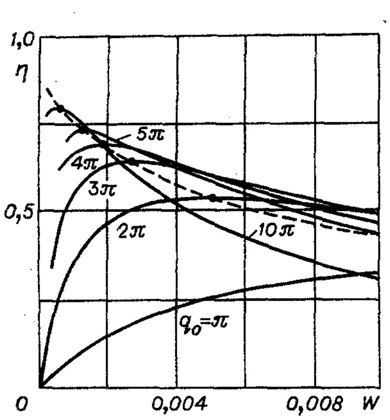

Fig. 5

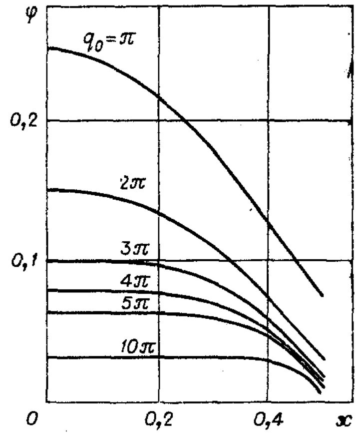

Fig. 6

The solid lines in Fig. 4 show $\eta(q_0)$ for various γ_1. The efficiency tends to the limit $\eta_\infty = \gamma_1$ for comparatively small q_0 (as (5.1) shows, the less γ_1, the lower the q_0 at which η_∞ is attained). This conduction unit differs substantially from an induction unit, in which [5] η approaches η_∞ at very small wavelengths (i.e., when numerous waves in the traveling field fit into the width of the plate). The maximum value of q_0 in Fig. 4 has been taken as 11π, while the asymptotic formula of (5.3) can be used to calculate η_0 for large q_0 for all $\gamma_1 \leqslant 0.9$.

It is of interest to examine how the efficiency of the system is dependent on H_0: to obtain the relationship we must equate the F_0 of (5.1) to $c_f/2\pi N$, as (4.1) implies, and determine the necessary value of γ_1; from γ_1 we get the result

$$\eta = W \frac{I_1(q_0)}{[W + (1/2) I_2(q_0)][W + I_1(q_0) + (1/2) I_2(q_0)]}, \quad W = c_f/2\pi N. \tag{5.5}$$

The parameter W relates the resistance coefficient c_f and the parameter N of the MHD interaction of (2.1) and is a universal parameter for describing the relationship. A parameter of this type was first used in [1], so W may be called the Way parameter.

We see from (5.5) that the $\eta(W)$ relationship is not monotone when the end-effects are considered: the efficiency falls to zero for $W = 0$ (i.e., for $H_0 = \infty$) and for $W = \infty$ ($H_0 = 0$), while for $W = W_*$, where

$$W_* = \sqrt{(1/2) I_2(q_0)[I_1(q_0) + (1/2) I_2(q_0)]}, \tag{5.6}$$

we get the maximum value of η:

$$\eta_{\max} = \frac{I_1(q_0)}{I_1(q_0) + I_2(q_0) + \sqrt{I_2(q_0)[2I_1(q_0) + I_2(q_0)]}}. \tag{5.7}$$

This constitutes the essential difference between these results and those of [1], which relate to an ideal model (the efficiency of an ideal model in the present planar geometry is $\eta_\infty = (1 + 2\pi q_0 W)^{-1}$ and increases monotonically as W decreases (H_0 increases)).

Therefore, a real conduction system resembles the induction system of [5] in having an optimum value of H_0, which is denoted by H_* and is dependent on q_0. For given q_0, a, and u_0 it is reasonable to increase H_0 only up to H_* (from the viewpoint of efficiency), since H_0 above H_* results in reduced efficiency.

The solid lines in Fig. 5 show the $\eta(W)$ of (5.5) for various q_0 up to 10π. The points show the $\eta_{\max}$ for the corresponding q_0, and the broken line at these points shows $\eta_{\max}(W)$, which is the result of eliminating q_0 from (5.6) and (5.7). Figure 4 shows the $W_*(q_0)$ of (5.6) (broken line). It is clear that W_* decreases as q_0 increases, and therefore H_* increases.

Figure 5 shows that it is necessary to work at small wave parameters in order to obtain large η or (for a given c_f) at large N. For example, $\eta \simeq 0.75$ is attained for $q_0 = 10\pi$, $W = 10^{-3}$. If we assume $c_f = 2 \cdot 10^{-3}$, we have $N = c_f/2\pi W = 1/\pi$, i.e., only by a large stretch of the imagination can the MHD interaction parameter be taken as satisfying (2.1). Therefore, the theory needs to be revised for the conditions under which high efficiency is attained because the bulk forces influence the velocity-field structure.

Figure 6 shows $\varphi(x)$ that define the optimum potential distribution on the electrodes for various q_0 from (4.4).

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COMPARISON OF PROSPECTIVE ENERGY SOURCES WITH THOSE IN USE

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UDC 621.548

In the Middle Ages the main energy source was the wind. Its energy was utilized by the sailing fleet and windmills. According to data of the Third International Symposium on Renewable Energy Sources (Turkey, 1977) the yearly per capita production of energy in Europe in the Middle Ages was 200 kWh. Windpower technology was particularly highly developed in Holland and Denmark.

At the present time half of all the world's power production is based on a very rapidly diminishing nonrenewable energy source — oil. The index of oil reserves, defined as the ratio of the proved recoverable reserves to the yearly extraction, is steadily declining. Before this index reaches values of the order of 10 or smaller, scientists must find a suitable economical equivalent, preferably from renewable sources, and succeed in becoming familiar with it on a large scale before power failures occur with all the consequent national economic shocks.

Coal reserves are practically inexhaustable in the present epoch, but difficult mining conditions keep its cost high.

The prime cost of oil in the world market and the selling price based on it have risen continuously in recent years. From 1970 to the present time the cost of 1 ton of crude oil has increased more than tenfold. The prime cost of 1 kWh of electric energy generated by a thermal electric power plant operating with fuel oil is 0.8-0.9 kopecks [1].

Water power, which makes up 19% of the total installed power of all electric power plants in the Soviet Union, cannot be considered a serious successor of oil energy resources [2]. The prime cost of 1 kWh generated by a hydroelectric power plant is 0.4 kopecks [3].

At the present time atomic power supplies a still smaller fraction of the total power supply. Taking account of the continuously increasing demands for purity of the environment, one can assume a continuous increase in the cost of 1 kWh of atomic energy. Its unprofitableness in stationary units will manifest itself more and more strongly with time. In addition to the purely monetary expressions of the high cost of atomic energy, its nonrenewability is further aggravated by the fact that it diverts a disproportionately large number of highly qualified workers from other understaffed branches of engineering where they might better serve the national economy.

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fikiki, No. 5, pp. 118-125, September-October, 1980. Original article submitted March 27, 1980.

0021-8944/80/2105-0673$07.50